2.8 ESTIMATION OF CLEAR-SKY RADIATION

The effects of the atmosphere in scattering and absorbing radiation are variable with time as atmospheric conditions and air mass change. It is useful to define a standard ''clear'' sky and calculate the hourly and daily radiation which would be received on a horizontal surface under these standard conditions.

Hottel (1976) has presented a method for estimating the beam radiation transmitted through clear atmospheres which takes into account zenith angle and altitude for a standard atmosphere and for four climate types. The atmospheric transmittance for beam radiation τ_b is G_{bn}/G_{on} (or G_{bT}/G_{oT}) and is given in the form

$$\tau_b = a_0 + a_1 \exp\left(\frac{-k}{\cos\theta_z}\right) \tag{2.8.1a}$$

The constants a_0, a_1, and k for the standard atmosphere with 23 km visibility are found from a_0^*, a_1^*, and k^*, which are given for altitudes less than 2.5 km by

$$a_0^* = 0.4237 - 0.00821(6 - A)^2 \tag{2.8.1b}$$

$$a_1^* = 0.5055 + 0.00595(6.5 - A)^2 \tag{2.8.1c}$$

$$k^* = 0.2711 + 0.01858(2.5 - A)^2 \tag{2.8.1d}$$

where A is the altitude of the observer in kilometers. (Hottel also gives equations for a_0^*, a_1^*, and k^* for a standard atmosphere with 5 km visibility.)

Correction factors are applied to a_0^*, a_1^*, and k^* to allow for changes in climate types. The correction factors $r_0 = a_0/a_0^*$, $r_1 = a_1/a_1^*$, and $r_k = k/k^*$ are given in Table 2.8.1. Thus, the transmittance of this standard atmosphere for beam radiation can be determined for any zenith angle and any altitude up to 2.5 km. The clear-sky beam normal radiation is then

$$G_{cnb} = G_{on}\tau_b \tag{2.8.2}$$

where G_{on} is obtained from Equation 1.4.1. The clear-sky horizontal beam radiation is

$$G_{cb} = G_{on}\tau_b \cos\theta_z \tag{2.8.3}$$

For periods of an hour, the clear-sky horizontal beam radiation is

$$I_{cb} = I_{on}\tau_b \cos\theta_z \tag{2.8.4}$$

Table 2.8.1 Correction Factors for Climate Types[a]

Climate Type	r_0	r_1	r_k
Tropical	0.95	0.98	1.02
Midlatitude summer	0.97	0.99	1.02
Subarctic summer	0.99	0.99	1.01
Midlatitude winter	1.03	1.01	1.00

[a]From Hottel (1976).

Example 2.8.1

Calculate the transmittance for beam radiation of the standard clear atmosphere at Madison (altitude 270 m) on August 22 at 11:30 AM solar time. Estimate the intensity of beam radiation at that time and its component on a horizontal surface.

Solution

On August 22, $n = 234$, the declination is 11.4°, and from Equation 1.6.5 the cosine of the zenith angle is 0.846.

The next step is to find the coefficients for Equation 2.8.1. First, the values for the standard atmosphere are obtained from Equations 2.8.1b to 2.8.1d for an altitude of 0.27 km:

$$a_0^* = 0.4237 - 0.00821(6 - 0.27)^2 = 0.154$$

$$a_1^* = 0.5055 + 0.00595(6.5 - 0.27)^2 = 0.736$$

$$k^* = 0.2711 + 0.01858(2.5 - 0.27)^2 = 0.363$$

The climate-type correction factors are obtained from Table 2.8.1 for midlatitude summer. Equation 2.8.1a becomes

$$\tau_b = 0.154 \times 0.97 + 0.736 \times 0.99 \ \exp\left(-0.363 \times \frac{1.02}{0.846}\right) = 0.62$$

The extraterrestrial radiation is 1339 W/m^2 from Equation 1.4.1. The beam radiation is then

$$G_{cnb} = 1339 \times 0.62 = 830 \ \text{W/m}^2$$

The component on a horizontal plane is

$$G_{cb} = 830 \times 0.846 = 702 \ \text{W/m}^2$$

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It is also necessary to estimate the clear-sky diffuse radiation on a horizontal surface to get the total radiation. Liu and Jordan (1960) developed an empirical relationship between the transmission coefficients for beam and diffuse radiation for clear days:

$$\tau_d = \frac{G_d}{G_o} = 0.271 - 0.294\tau_b \tag{2.8.5}$$

where τ_d is G_d/G_o (or I_d/I_o), the ratio of diffuse radiation to the extraterrestrial (beam) radiation on the horizontal plane. The equation is based on data for three stations. The data used by Liu and Jordan predated that used by Hottel (1976) and may not be entirely consistent with it; until better information becomes available, it is suggested that Equation 2.8.5 be used to estimate diffuse clear-sky radiation, which can be added to the beam radiation predicted by Hottel's method to obtain a clear hour's total. These calculations can be repeated for each hour of the day, based on the midpoints of the hours, to obtain a standard clear day's radiation H_c.

Example 2.8.2

Estimate the standard clear-day radiation on a horizontal surface for Madison on August 22.

Solution

For each hour, based on the midpoints of the hour, the transmittances of the atmosphere for beam and diffuse radiation are estimated. The calculation of τ_b is illustrated for the hour 11 to 12 (i.e., at 11:30) in Example 2.8.1, and the beam radiation for a horizontal surface for the hour is 2.53 MJ/m^2 (702 W/m^2 for the hour).

The calculation of τ_d is based on Equation 2.8.5:

$$\tau_d = 0.271 - 0.294(0.62) = 0.089$$

Next G_{on}, calculated by Equation 1.4.1, is 1339 W/m^2. Then G_o is $G_{on} \cos\theta_z$ so that

$$G_{cd} = 1339 \times 0.846 \times 0.089 = 101 \text{ W/m}^2$$

Then the diffuse radiation for the hour is 0.36 MJ/m^2. The total radiation on a horizontal plane for the hour is $2.53 + 0.36 = 2.89$ MJ/m^2. These calculations are repeated for each hour of the day. The result is shown in the tabulation, where energy quantities are in megajoules per square meter. The beam for the day H_{cb} is twice the sum of column 4, giving 19.0 MJ/m^2. The day's total radiation H_c is twice the sum of column 7, or 22.8 MJ/m^2.

		I_{cb}				
Hours	τ_b	Normal	Horizontal	τ_d	I_{cd}	I_c
11–12, 12–1	0.620	2.99	2.52	0.089	0.36	2.89
10–11, 1–2	0.607	2.93	2.33	0.092	0.35	2.69
9–10, 2–3	0.580	2.79	1.97	0.100	0.34	2.31
8–9, 3–4	0.530	2.56	1.46	0.115	0.32	1.78
7–8, 4–5	0.444	2.14	0.88	0.140	0.28	1.15
6–7, 5–6	0.293	1.41	0.32	0.185	0.20	0.53
5–6, 6–7	0.150	0.72	0.03	0.227	0.05	0.07

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A simpler method for estimating clear-sky radiation by hours is to use data for the ASHRAE standard atmosphere. Farber and Morrison (1977) provide tables of beam normal radiation and total radiation on a horizontal surface as a function of zenith angle. These are plotted in Figure 2.8.1. For a given day, hour-by-hour estimates of I can be made, based on midpoints of the hours.