Rain, Snow, and Spring Runoff Revisited

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More than a decade ago, Bartlett[1] published in this journal an article in which he criticized the media for repeating every spring "the standard springtime story that warm rain falling on snow [in the Rocky Mountains] could 'trigger' major flooding." He then went on to show that the amount of snow that rain of any reasonable temperature could melt would enhance runoff by not more than about 10%. Bartlett's analysis is correct insofar as it addresses why warm rainfall per se is not responsible for rapid snowmelt. Yet the observations that rainfall and rapid runoff are sometimes correlated is valid. Explanations for this correlation, however, are another matter. Yet it is fairly well known to meteorologists that warm fogs are notorious for causing snow to disappear more rapidly than it would otherwise.

Throughout this article, by warm I mean a temperature greater than 0°C, and by snow melting I do not mean settling or sublimating. A snowpack can visibly shrink because of melting, settling, or sublimating. Whenever depths are expressed they are depths of water, not of snow. A snowpack 10 cm deep will melt to yield only 1 to 3 cm of liquid water (water equivalent). Whereas Bartlett explained what does not happen, I shall explain what does (or at least can) happen.

For completeness, the following is a summary of Bartlett's arguments. When a given mass M_w of water is cooled from some initial temperature t_a (°C) to 0°C, its decrease in enthalpy is

$$\Delta H_w = -c_p M_w t_a \tag{1}$$

where c_p is the heat capacity (at constant pressure) per unit mass of water. Enthalpy H is defined as the sum of the thermodynamic internal energy U and the product pV, where p is pressure and V is volume. In an adiabatic process (one in which a system does not change its energy by virtue of interaction with its surroundings at a different temperature) at constant pressure, enthalpy is conserved. Many processes in nature occur at constant pressure rather than at constant volume. For example, when ice cubes are dropped into warm water in an insulated container, the subsequent melting of the ice and cooling of the water is essentially a constant enthalpy rather than a constant energy process.† When a given mass M_i of ice melts, its enthalpy increase is

$$\Delta H_i = \lambda_f M_i \tag{2}$$

where λ_f is the enthalpy (latent heat) of fusion of ice. When ice and water are mixed adiabatically at constant pressure, total enthalpy is conserved. Thus, by setting the sum of these two enthalpy changes [Eqs. (1) and (2)] equal to zero, we obtain

$$\frac{M_i}{M_w} = \frac{c_p t_a}{\lambda_f} \tag{3}$$

For water near 0°C, c_p = 4218 J/kg K, and $\lambda_f = 3.34 \times 10^5$ J/kg. With these values in Eq. (3) we obtain

$$M_i \approx M_w \frac{t_a}{80} \tag{4}$$

This is the *maximum* amount of ice that a given mass of water at a temperature t_a can melt. In obtaining Eq. (3) I appealed to conservation of enthalpy, encouraged by seeing enthalpy arguments in a recent article in this journal.[2] The time is long overdue for enthalpy to replace "quantity of heat" and other archaic survivors of the caloric theory. As long ago as 1950, O'Leary argued that "I think we ought to rid ourselves of the ideas of heat, flow of heat, and flow of energy...thermal effects can be accurately described in terms of *enthalpy*, *thermal transfer*, and *energy transfer* with no connotation in regard to a fluid." But, alas, the writers of textbooks read only other textbooks, so O'Leary's sensible and cogent suggestions were of no avail. Virtually every textbook makes a point of asserting that there is no such thing as the amount of heat in a body, then proceeds as if there were. For more on this, see Refs. 4 and 5.

When rain falls on a deep snowpack in early spring, air temperatures are not likely to be more than a few degrees above freezing. Of course air temperatures in summer in the Rocky Mountains can rise to much higher values, but when this occurs snow on the ground has long since disappeared. Thus a reasonable value for t_a for rain during a rainfall-stimulated rapid spring snowmelt is perhaps 1 to 5°C. We therefore conclude, along with Bartlett, that it would take a staggering amount of rain to melt an appreciable amount of snow, and such rain would itself be the cause of flooding, not the enhanced snowmelt. For example, 40 cm of rain at 2°C falling on snow could increase the *total* runoff (rainfall + snowmelt) by only 1 cm (water equivalent). All is well until we reflect on the fact that warm fogs also sometimes result in rapid snowmelt, and such fogs do not themselves contribute appreciably to runoff. This observation is a clue to the physical origins of enhanced snowmelt runoff.

It is probably safe to make the sweeping statement that everyone has experienced the consequences of evaporative cooling. And it is almost as safe to say that no one has directly experienced the logical inverse of this process: condensational heating. If (net) evaporation is a cooling process, (net) condensation must be a heating process. Suppose that an amount of water vapor M_v (at 0°C) condenses to form liquid water. The decrease in enthalpy of this water vapor is

$$\Delta H_v = -M_v \lambda_v \tag{5}$$

where λ_v is the enthalpy (latent heat) of vaporization. If we set this enthalpy decrease plus the enthalpy increase [from Eq. (2)] of ice upon melting equal to zero, we obtain

$$M_i = M_v \frac{\lambda_v}{\lambda_f} \tag{6}$$

The enthalpy of vaporization is about 2.5×10^6 J/kg. This value and the previous value for the enthalpy of fusion yields

$$M_i = 7.5 M_v \tag{7}$$

Thus each gram of water vapor that condenses on snow can melt 7.5 grams of ice. From Eqs. (4) and (7) it follows that the ratio of the mass of ice that can be melted by a given mass of water vapor when it condenses to the amount of ice that can be melted by the same mass of liquid water when it cools from t_a to 0°C is

$$\frac{600}{t_a} \tag{8}$$

Given that t_a for our problem is around 1 to 5°C, we conclude from Eq. (8) that a small amount of net condensation of water vapor onto snow can melt vastly more—hundreds of times more—ice than can warm rain upon cooling to the freezing point. Energetically, a small amount of net condensation goes a long way. And the amount of rain necessary to melt an appreciable amount of snow is perceptible, whereas the amount of vapor deposition required to melt the same amount of snow is imperceptible.

There will be net condensation onto a snowpack if the partial pressure (or vapor density) of water vapor in the air above the snowpack exceeds the partial pressure (vapor density) of water vapor at the snow surface. The temperature of sweat on human skin can rise and fall because of physiological processes and changes in ambient temperature and humidity. But snow, unlike human flesh, is constrained to have a temperature less than or equal to 0°C. In turn, this constrains the vapor pressure at the surface of snow (ice) to be less than or equal to 6.11 mbar (1 bar = 10^5 Pa). But the partial pressure of water vapor in the air well above the snowpack is not so constrained. In particular, if this partial pressure rises above 6.11 mbar, there will be a net vapor flux into the snowpack with concomitant condensational heating, and hence snowmelt.

To determine the conditions under which this can happen, consider the following expression for the temperature dependence of the saturation (or equilibrium) vapor pressure e_s of water:

$$\ln\left(\frac{e_s}{e_{so}}\right) = 19.39 - \frac{5393}{T} \tag{9}$$

where T is absolute temperature and e_{so} = 8.718 mbar is the saturation vapor pressure at 5°C. This expression is sufficiently accurate for our purposes. It was obtained by integrating the Clapeyron equation[6] with a constant enthalpy of vaporization equal to its value at 5°C. The relative humidity r of moist air at an absolute temperature T_a is defined as the actual vapor pressure e (partial pressure of water vapor) relative to what it would be if the vapor were in equilibrium with liquid (or solid) water at this temperature:

$$r = \frac{e}{e_s(T_a)} \tag{10}$$

There will be a net vapor flux into snow for all those values of T_a and r such that $e > 6.11$ mbar. According to Eqs. (9) and (10), for an air temperature of 1°C, there will be net condensation into the snowpack if the relative humidity (expressed as a percent) exceeds 93%. For an air temperature of 2°C, there will be net condensation if the relative humidity exceeds 85%. And for an air temperature of 3°C, there will be net condensation if the relative humidity exceeds 78%. None of these requirements on humidity is especially stringent: we do not have to stretch the limits of the possible to posit relative humidities in excess of 95% during rain or fog, the function of which is not to directly melt snow but to provide the high relative humidities needed for net condensation, hence condensational melting. All else being equal (wind speed and relative humidity), condensation of water vapor will increase dramatically with air temperature because of the exponential dependence of vapor pressure on temperature [Eq. (9)].

What observational evidence can be brought to bear on the question of enhanced runoff snowmelt because of condensation of water vapor on snow? To answer this question I wrote to Sam Colbeck at the Cold Regions Research and Engineering Laboratory in Hanover, New Hampshire. The editor of Ref. 7 and an authority on snow, he informed me that "the most rapid snowmelt that I have observed occurs due to latent heat exchange. On the Blue Glacier [Olympic National Park] we frequently would lose snow like crazy on days of heavy overcast and 10-mph winds with temperatures about 40°F. The loss rate was faster than on sunny and warm days." Amback and Kirchlechner[8] give nomographs for determining meltwater from snow resulting from convectional and condensational (latent) heating. At 6°C with a wind speed of 10 m/s and a relative humidity of 100%, the melt rate is 25 mm/day (water equivalent) just from condensational heating. Whether this is sufficient to "trigger massive flooding" depends on what one means by "massive." Nevertheless, it remains that a prolonged warm rain, hence prolonged high humidity, can be the indirect cause of as much runoff as that contributed by the rain alone. This is not a consequence of rain per se but of the high humidities associated with rain. It is invisible water vapor that melts snow, not visible liquid water.

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†[*Editor's Note*: For the system of the ice cube alone, the first law of thermodynamics requires that: $Q = \Delta U + p\Delta V$. The heat Q, coming from the warm water, changes the internal energy U of the ice cube. This energy is used in changing the state from ice to water, without change in temperature. The *contraction* as the ice turns into water makes $p\Delta V$ negative, thus decreasing the Q necessary. Let us see what the difference is between enthalpy and internal energy in this case. Since $H = U + pV$, $\Delta H = \Delta U + p\Delta V + V\Delta p \rightarrow \Delta U + p\Delta V$ at constant pressure. For the change of state of ice to water, $\Delta U = 80$ cal/g = 330 J/g. As 1 g of ice turns to 1 g of water at atmospheric pressure, $\Delta V = 0.1\ \text{cm}^3$ and $p\Delta V = (1 \times 10^5\ \text{N/m}^2)(1 \times 10^{-7}\ \text{m}^3) = 1 \times 10^{-2}$ J. Compare this small effect with ΔU. Evidently for transformation of solid to liquid at constant pressure, there is very little difference between the change in internal energy and the change in enthalpy. When a gas is generated in a transformation, ΔV can be large, and then the distinction between U and H becomes more important.]